

Calculus Practice SAC

Worked Solutions

VCE Methods Units 1&2

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Section A – Technology-free

Question 1a

3 marks

The velocity is $v(t) = 2pt + 3qt^2$, which is already 0 at $t = 0$. The train is at B and at rest when $t = 30$.

$$900p + 27\,000q = 450 \quad [1M]$$

$$60p + 2700q = 0 \quad [1M]$$

$$p = -45q$$

$$900(-45q) + 27\,000q = 450$$

$$-13\,500q = 450$$

$$q = -\frac{1}{30}$$

$$p = \frac{45}{30}$$

$$= \frac{3}{2}$$

[1A]

Question 1b

2 marks

The speed is greatest where the acceleration is zero.

$$v(t) = 3t - \frac{t^2}{10}$$

$$a(t) = 3 - \frac{t}{5} \quad [1M]$$

$$t = 15$$

$$v(15) = 45 - 22.5$$

$$= 22.5 \text{ m/s}$$

$$22.5 \times 3.6 = 81 \text{ km/h} \quad [1A]$$

Question 1c

1 mark

$$\frac{450}{30} = 15 \text{ m/s}$$

[1A]

Question 1d

1 mark

$$a(20) = 3 - \frac{20}{5}$$

$$= -1 \text{ m/s}^2$$

[1A]

Question 1e

2 marks

$$3t - \frac{t^2}{10} = 20$$

$$t^2 - 30t + 200 = 0$$

$$(t - 10)(t - 20) = 0$$

[1M]

$$t = 10 \text{ s or } 20 \text{ s}$$

[1A]

Section B – Technology-active

Question 2a

1 mark

Multiply both sides by -30 .

$$\frac{3t^2}{2} - \frac{t^3}{30} = 126$$

$$45t^2 - t^3 = 3780$$

$$t^3 - 45t^2 + 3780 = 0$$

[1A]

Question 2b

2 marks

$$g(t) = t^3 - 45t^2 + 3780$$

$$g'(t) = 3t^2 - 90t$$

$$g(12) = -972$$

$$g'(12) = -648$$

$$t_1 = 12 - \frac{-972}{-648}$$

$$= 10.5$$

[1M]

[1A]

Question 2c

1 mark

Continue Newton's method with technology: $t_2 \approx 10.4615$, $t_3 \approx 10.4615$.

$$t \approx 10.46 \text{ s}$$

[1A]

Question 2d

3 marks

The back of the train passes the crossing when the front is 151 m from A. Solve with technology.

$$\frac{3t^2}{2} - \frac{t^3}{30} = 151$$

$$t \approx 11.6557$$

[1M]

$$10.4615 - 7 = 3.4615$$

$$11.6557 + 3 = 14.6557$$

[1M]

$$14.6557 - 3.4615 = 11.1942$$

$$\approx 11 \text{ s}$$

[1A]

Question 2e

1 mark

$$v(11.6557) \approx 21.3816 \text{ m/s}$$

$$21.3816 \times 3.6 \approx 77.0 \text{ km/h}$$

[1A]

Question 3a

2 marks

$$pT^2 + qT^3 = 450$$

$$2pT + 3qT^2 = 0$$

Solve with technology.

$$p = \frac{1350}{T^2}$$

$$q = -\frac{900}{T^3}$$

[1M]

[1A]

Question 3b

2 marks

$$a(t) = 2p + 6qt$$

$$= \frac{2700}{T^2} - \frac{5400t}{T^3}$$

[1M]

The acceleration is linear in t , so its greatest magnitude is at an endpoint of $[0, T]$.

$$a(0) = \frac{2700}{T^2}$$

$$a(T) = -\frac{2700}{T^2}$$

[1A]

Question 3c

2 marks

$$\frac{2700}{T^2} \leq 3.6$$

$$T^2 \geq 750$$

$$T \geq 5\sqrt{30}$$

$$\approx 27.39$$

[1M]

 $T = 27$ gives a greatest acceleration of about 3.704 m/s^2 , which is too large.

$$T = 28 \text{ s}$$

[1A]

Question 3d

2 marks

The acceleration is zero halfway through the trip.

$$v\left(\frac{T}{2}\right) = pT + \frac{3qT^2}{4}$$

$$= \frac{675}{T}$$

$$\frac{675}{28} \approx 24.107 \text{ m/s}$$

$$24.107 \times 3.6 \approx 87 \text{ km/h}$$

[1M]

[1A]

Question 4a

2 marks

$$v(t) = -0.3t - 0.01t^2 + c$$

[1M]

$$v(0) = 25, \text{ so } c = 25.$$

$$v(t) = 25 - 0.3t - 0.01t^2 \text{ m/s} \quad [1A]$$

Question 4b

2 marks

$$25 - 0.3t - 0.01t^2 = 0$$

$$t^2 + 30t - 2500 = 0 \quad [1M]$$

Take the positive solution.

$$t = -15 + 5\sqrt{109} \quad [1A]$$

$$\approx 37.20 \text{ s}$$

Question 4c

2 marks

Let $s(t)$ be the distance travelled since the brakes were applied, so $s(0) = 0$.

$$s(t) = 25t - 0.15t^2 - \frac{t^3}{300} \quad [1M]$$

$$s(37.2015) \approx 550.83 \quad [1A]$$

$$\approx 551 \text{ m}$$

Question 4d

2 marks

$$25t - 0.15t^2 - \frac{t^3}{300} = \frac{550.83}{2}, \quad 0 \leq t \leq 37.2015 \quad [1M]$$

Solve with technology.

$$t \approx 12.1 \text{ s} \quad [1A]$$

Question 5a

2 marks

$$\frac{3t^2}{2} - \frac{t^3}{30} = 16t$$

$$t^2 - 45t + 480 = 0 \quad [1M]$$

Both solutions lie in $(0, 30]$.

$$t = \frac{45 - \sqrt{105}}{2} \text{ s or } \frac{45 + \sqrt{105}}{2} \text{ s} \quad [1A]$$

Question 5b

2 marks

The train is ahead between the two times in part a. The gap $x(t) - 16t$ is greatest where the train's velocity is 16 m/s. The other solution, $15 - \sqrt{65}$, is before the first time in part a.

$$3t - \frac{t^2}{10} = 16 \quad [1M]$$

$$t = 15 + \sqrt{65} \approx 23.06$$

$$\begin{aligned}
 x(23.06) - 16(23.06) &\approx 19.94 \\
 &\approx 20 \text{ m}
 \end{aligned}
 \qquad [1A]$$

Question 5c

3 marks

Divide both sides of $x(t) = ut$ by t , since $t > 0$.

$$u = \frac{3t}{2} - \frac{t^2}{30}, \quad 0 < t \leq 30 \qquad [1M]$$

The right side is a quadratic in t with turning point at $t = 22.5$. It increases from 0 to $\frac{135}{8}$, then decreases to 15 at $t = 30$.

$$\begin{aligned}
 \frac{3(22.5)}{2} - \frac{22.5^2}{30} &= \frac{135}{8} \\
 \frac{3(30)}{2} - \frac{30^2}{30} &= 15
 \end{aligned}
 \qquad [1M]$$

A horizontal line meets this graph exactly once below 15 or at the maximum.

$$0 < u < 15 \text{ or } u = \frac{135}{8} \qquad [1A]$$